Journal of Visualization, Vol. 1, No. 2 (1998) 153-160

Three-dimensional Particle Cluster Tracking Algorithm using Affine Transformation

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Received 10 March 1998.
Revised 26 June 1998.

***Abstract*:** In Particle Image Velocimetry (PIV), the cross correlation tracking technique is widely used to analyze the particle images. The technique assumes that the fluid motion, within small regions of the flow field, is given by parallel movements over short time intervals. However, actual flow fields may have some distorted motion,such as rotation and shear. Therefore, if the distortion of the flow field is not negligible, the fluid motion can not be tracked well using the cross correlation technique.

The author proposed a new particle tracking technique, based on the particle cluster matching using linear Affine Transformation. The algorithm can be applied to flow fields which exhibit characteristics such as rotation and shear. The deformation of the cluster pattern is expressed by the linear Affine Transformation. The parameter of the transformation can be determined using the least square technique from the particle positions.

The effectiveness of the tracking techniques, including 3D cross correlation, Spring Model and Affine Transformation, were evaluated with synthetic data of three-dimensional flow field. The cross correlation technique could be applicable to the small deformation cases. When the deformation of particle pattern between two images are very large, the pattern deformation could not be expressed by the Affine Transformation, i.e., linear transformation, resulting in mis-tracking. However, the Spring Model technique was found to be more effective even in the larger deformation condition, because the Spring Model does not assume the linear transformation.

***Keywords*:** particle imaging velocimetry, Spring Model, Affine Transformation, three-dimensional measurement, pattern matching.

1. Introduction

Particle Image Velocimetry (PIV) is a superior measurement technique for studying fluid flows. Its primary advantage is the ability to capture spatial velocity distributions simultaneously and noninvasively.

The fluid flow is visualized by seeding with small particle tracers. Then the flow visualization images are analyzed to obtain the velocity distribution of the flow. The cross correlation technique is widely used to calculate the seed movement (Adrian, 1991; Hassan et al., 1992; Hassan et al., 1994). The technique assumes that the fluid motion, within small regions of the flow field, is given by parallel movements over short time intervals. However, actual flow fields may have some distorted motion, such as rotation and shear. Therefore, if the distorted motions are not negligible, the cross correlation technique can not track the flow.

The author proposed a Spring Model particle tracking technique (Okamoto et al., 1995). The algorithm can be applied to flow fields which exhibit characteristics such as rotation and shear. In the Spring Model technique, the smallest deformation pattern, i.e., Spring force pattern, was selected as the corresponding pattern. However, the information of pattern deformation was not used in the pattern tracking, since the Spring force, F, had less physical meanings.

Dracos et al. (1995) derived the Affine transformation for tracking the 3D Laser Induced Fluorescence (LIF) data. They calculated the coefficients of the Affine transformation based on the 3D deformation of the LIF pattern, resulting in the vorticity tensor and the stress tensor. Because the LIF data filled in the whole volume, it is easy to calculate these tensor. However, in the PIV images, the particles were sparsely distributed, resulting in the difficulties to apply their technique to PIV.

Ishikawa et al. (1996) also proposed a particle pattern matching technique. The vector gradient tensor for the fluid motion was calculated using the particle positions in the cluster. Then the cross-correlation technique was applied by considering the tensor. They demonstrated the technique in two-dimensional field.

In this study, an improved technique of three-dimensional particle tracking was proposed, based on the particle cluster matching. The three-dimensional tensor was obtained using the particle position information, with assuming the linear Affine Transformation. Then, the cluster matching was performed by the residue of tensor calculation. The stress tensor was directly calculated in the Affine Transformation.

In this study, to discuss the effectiveness of the three-dimensional particle cluster tracking algorithm, synthetic three-dimensional flow fields were analyzed, using the 3D Cross-correlation technique, the Spring Model and the new Affine Transformation technique.

2. Particle Cluster Matching with Affine Transformation

In the cross-correlation technique, the cross-correlation between two images of particles was calculated. The new technique is based on the pattern matching of the particle cluster between two consecutive images. The particle cluster with 15 to 20 particles was considered. The pattern deformation in the Affine Transformation is expressed as follows,

$$\mathbf{X}_1 = \mathbf{A}\mathbf{X}_0 + \mathbf{X}_d \tag{1}$$

That is,

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{pmatrix} x_d \\ y_d \\ z_d \end{pmatrix} \tag{2}$$

where $\mathbf{X}_0$, (x_0, y_0, z_0) and $\mathbf{X}_1$, (x_1, y_1, z_1) denote the coordinate system in the first and second images, respectively. A denotes the deformation matrix and X_d is the displacement vector. The origin of the coordinate was assumed to be the center particle of the selected cluster in the first image. From the displacement vector between the first and second images, (x_d, y_d, z_d), the velocity is obtained. When the matrix **A** is the unit tensor, the deformation was only the parallel motion.

Because of the continuity, the relation, $\nabla\vec{u} = 0$, should be satisfied in the Affine transformation, i.e., $a_{11} + a_{22} + a_{33} = 3$. The deformation matrix, **A**, is re-written as follows,

$$\begin{aligned} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} &= \begin{pmatrix} 1+\sigma_{11} & \sigma_{12}+\tau_{12} & \sigma_{13}+\tau_{13} \\ \sigma_{12}-\tau_{12} & 1+\sigma_{22} & \sigma_{23}+\tau_{23} \\ \sigma_{13}-\tau_{13} & \sigma_{23}-\tau_{23} & 1+\sigma_{33} \end{pmatrix} \\ &= \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix} + \begin{pmatrix} 1 & \tau_{12} & \tau_{13} \\ -\tau_{12} & 1 & \tau_{23} \\ -\tau_{13} & -\tau_{23} & 1 \end{pmatrix} \end{aligned} \tag{3}$$

where $\sigma_{11} + \sigma_{22} + \sigma_{33} = 0$.

The 1st and 2nd term denote the shear and rotational tensor, respectively. The tensor **A** has 8 independent parameters. These parameters are determined using least square technique. Therefore, the particle cluster should have more than 9 particles. The residual, $R_{i,j}$, is defined as,

$$R_{i,j}=\sum_{k}^{N}\left|\mathbf{X}_{j,(k)}-\mathbf{A}\mathbf{X}_{i,(k)}-\mathbf{X}_{d,(i,j)}\right|^2 \tag{4}$$

where $X_{j,(k)}$ and $X_{i,(k)}$ denote the location of particle (k) in the particle cluster (i) for the first image and that (j) for the second image, respectively.

In this study, the residual, $R_{i,j}$, was taken as the pattern matching parameter. When the cluster in the second image does not correspond to the original cluster in first image, the particle pattern deformation will be quite large, resulting in larger residual value.

The least square technique just gives the arguments which show the least residual value. That means, the deformation parameter does not correspond to the residual value. The deformation parameter could not be used as the pattern matching parameter, because the deformation pattern might be small in non-corresponding clusters. For example, even though the obtained Affine matrix was unit matrix, i.e., no deformation, the residual value could be very large when the clusters did not correspond. So if the deformation parameter in the Affine matrix was used as the pattern matching parameter, there will be several of incorrect matching.

For the corresponding clusters, the residual value is almost zero, although the deformation might be very large. Therefore, the corresponding particle clusters could be determined using the information of residual value, $R_{i,j}$. By applying the present technique, the corresponding particle clusters were determined uniquely.

In the calculation, the information of stress tensor was directly obtained. In this study, the scalar stress was defined as,

$$S=\sigma_{12}^2+\sigma_{23}^2+\sigma_{13}^2+\frac{1}{2}(\sigma_{11}^2+\sigma_{22}^2+\sigma_{33}^2) \tag{5}$$

The scalar stress denotes the effects of the pattern deformation. Therefore, in this study, the scalar stress (S) is used as deformation parameter.

When the three-dimensional particle positions were obtained from the HPIV technique (Barnhart et al., 1994) or stereo camera technique, the particle cluster matching was carried out using the information on particle locations. For the target particle cluster in the first image, the residual of the Affine Transformation for every possible target particle cluster in the second image was calculated. Then, the smallest residual particle cluster corresponds to the target particle cluster in the first image.

Using this technique, even when rotation and shear are applied to the flow, the particles can be correctly tracked.

3. Simulation and Discussion

In order to discuss the effectiveness of the proposed technique, velocity distributions were reconstructed from synthetic particle data.

The three-dimensional flow field was calculated by a numerical simulation code. Using the simulated flow, the movement of the particles was calculated. Then, the flow was reconstructed from the particle information, using the present technique. To compare the results, the flow was also reconstructed using the three-dimensional cross correlation technique (Hassan et al., 1994) and the Spring Model technique (Okamoto et al., 1995) with the same particle information.

Figure 1 shows the simulated three-dimensional laminar flow field with large circulating flow calculated by a laminar Finite Element Method (FEM) code. The size of the images is 200×200×200 pixel. Large shear existed at the entrance and bottom region, causing the particle tracking to be very difficult.

As parameters of the simulation, the number of particles (n) and the average displacement (d_m) were taken into account. When the number of particles increased, the average distance between the particles decreased, resulting in the deformation effects to be small. The size of the particle cluster is also the function of (n), because the number of particles in one cluster was fixed to be 15 to 20. The average displacement was also a parameter of the deformation. In this simulation, because the correct particle pairs were known, the effectiveness of the technique was quantitatively evaluated. The correct ratio was defined as the number of correctly detected vectors to number of all particles (vectors), i.e., $R_c = n_{\mathrm{correct}}/n$.

Figures 2 and 3 show the example of the reconstructed 3D flow field by the 3D cross correlation and Spring

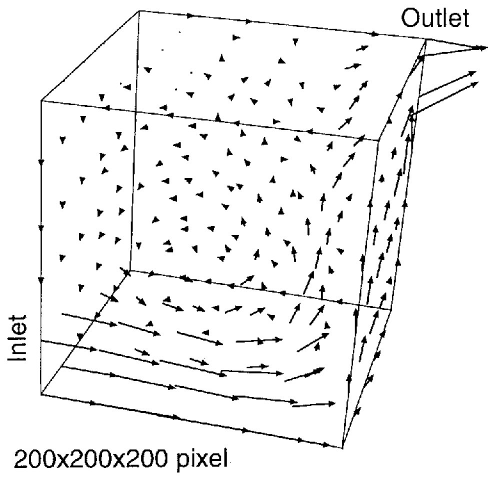

Fig. 1. Reference three-dimensional flow.

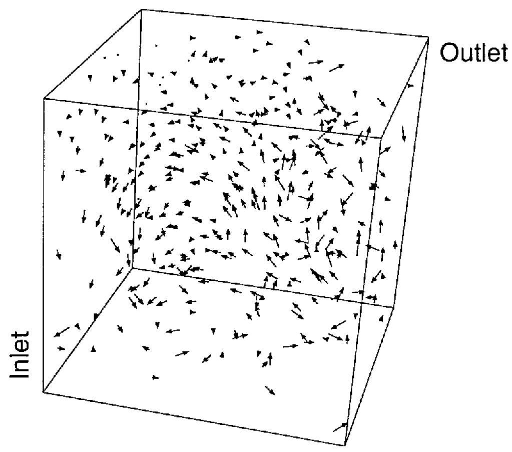

Fig. 2. Three-dimensional velocity distributions reconstructed by 3D cross correlation (Hassan et al., 1994).

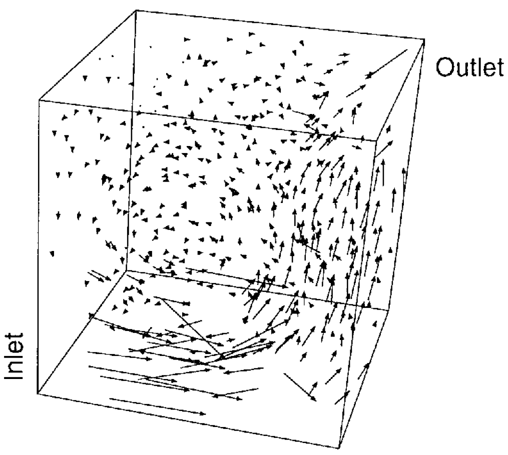

Fig. 3. Three-dimensional velocity distributions reconstructed by Spring Model (Okamoto et al., 1995).

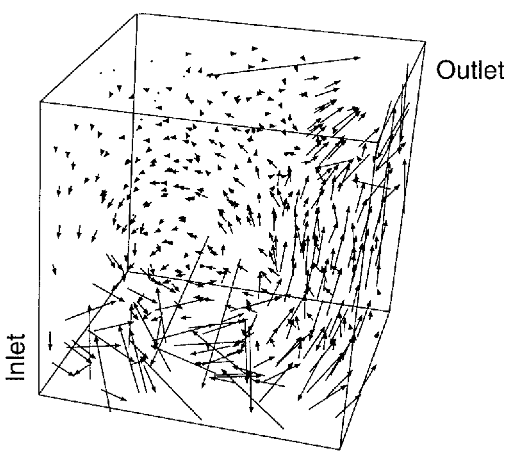

Fig. 4. Three-dimensional velocity distributions reconstructed by Affine Transformation.

Model technique, respectively. The cross correlation technique could not reconstruct the high shear region. While, the Spring Model can reconstruct the flow field well. Figure 4 shows the example of the reconstructed flow field by the Affine Transformation technique. In this case, almost 80% vectors were correctly calculated. The new technique can reconstruct the flow field. In Figs. 3 and 4, there were several large erroneous vectors, because of too large displacements. These large vectors could be easily removed with applying the cleaning process (e.g., Kimura et al., 1993).

Figure 5 shows the relationship between the correct ratio (R_c) and average displacement (d_m). With increasing average displacement, the correct ratio decreased, because of larger deformation. The cross correlation technique correctly reconstructs the flow field when the average displacement is small enough ($d_m < 2$). However, in the larger displacement case, the deformation caused larger mis-tracking, resulting in small correct ratio. The Spring Model also correctly reconstructs the flow field in the small displacement case. Since the effects of non-linear distortion of the particle cluster were taken into account in the Spring Model, the correct ratio was relatively high even in the larger displacement cases. The Affine Transformation technique had the similar tendency to the Spring Model with relatively lower level. However, the correct ratio in the small distortion case was worse than that of cross correlation. In the Affine Transformation technique, one cluster should be consist of at least 8 particles to determine the matrix A as show in Eq. (3). While, that of Spring Model needed only 3 or 4 particles. Therefore, in the sparse particle region, several particles did not contribute to any cluster. They were counted as the wrong vector in Fig. 5, resulting in the correct ratio to be lower. For the Affine Transformation technique, more dense particle condition was required.

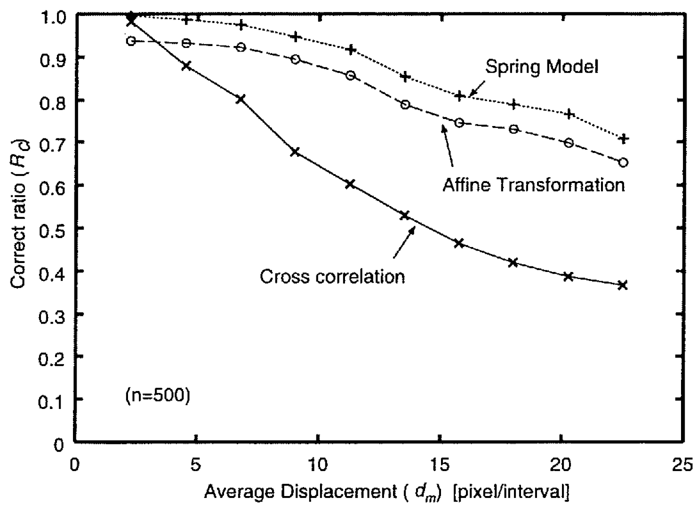

Fig. 5. Relationship between the average displacement and correct ratio.

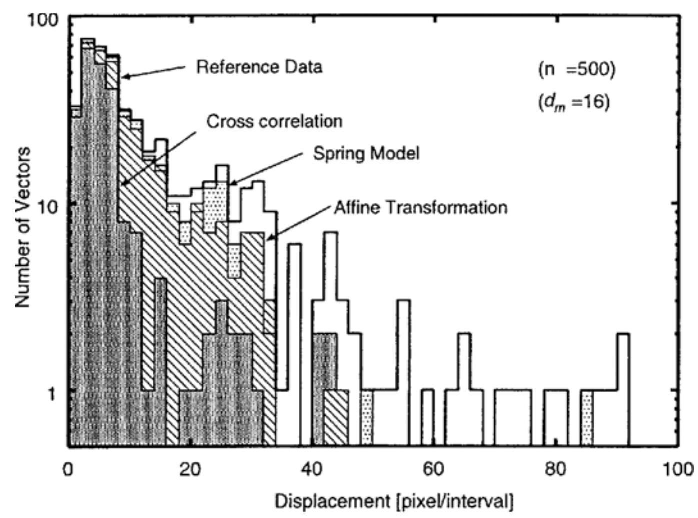

Fig. 6. Effects of displacement on tracking efficiency.

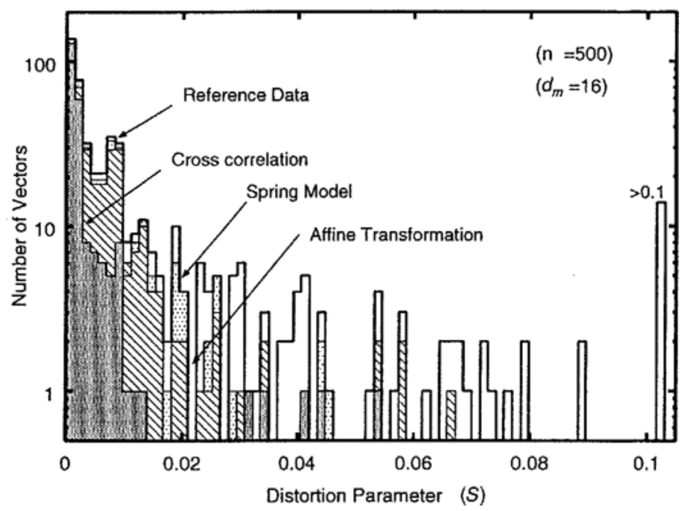

Fig. 7. Effects of distortion on tracking efficiency.

In Fig. 5, the correct ratio was calculated for all of the vector in the flow field. However, the displacement and distortion were function of the location in the flow field. Figures 6 and 7 show the histogram of the vectors in the simulated flow field. In this simulation, 500 particles were inserted into the flow field with the average displacement of 16 pixel. Figure 6 shows the histogram for the vector amplitude, i.e., particle displacement. The cross correlation could correctly track the particles in the lower displacement. The Spring Model and Affine Transformation could track the particles with relatively larger displacement. Figure 7 shows the histogram for the distortion parameter (S in Eq. (5)). Tracking efficiency of the cross correlation technique rapidly decreased with increasing the distortion parameter. While, the efficiency of the Spring Model and Affine Transformation decreased slowly. They could be applicable to the relatively larger distortion.

In these figures, the difference between the Spring Model and Affine Transformation was not clear. To evaluate the effectiveness, all of the tracking efficiencies (correct ratio) for these technique were plotted as shown in Figs. 8 and 9. The data distributed widely, however, the correct ratio of Spring Model was relatively better than that of Affine Transformation. The difference was mainly caused by the difference of acceptable distortion, i.e., non-linear and linear distortion. Since the Affine Transformation was a linear transformation, the deformation should be expressed by the linear equation. However, in the present case, the deformation was too large to assume the linear deformation, resulting in the error of Affine Transformation. Therefore, the larger deformation could not be tracked. The Spring Model was found to be the better tracking technique under the larger deformation, since it did not assume the linear deformation.

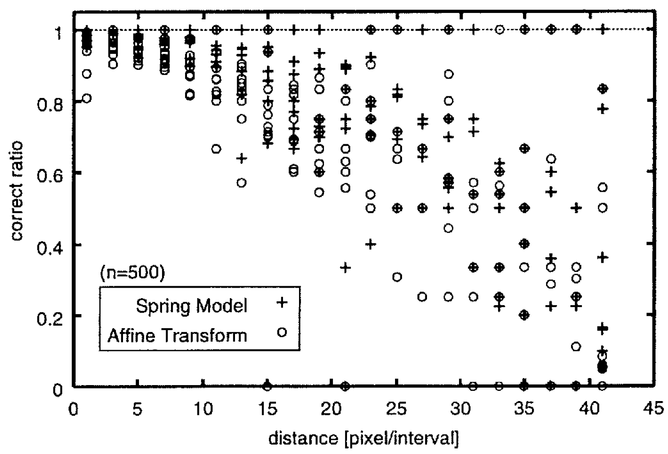

Fig. 8. Correct ratio with displacement for two new techniques.

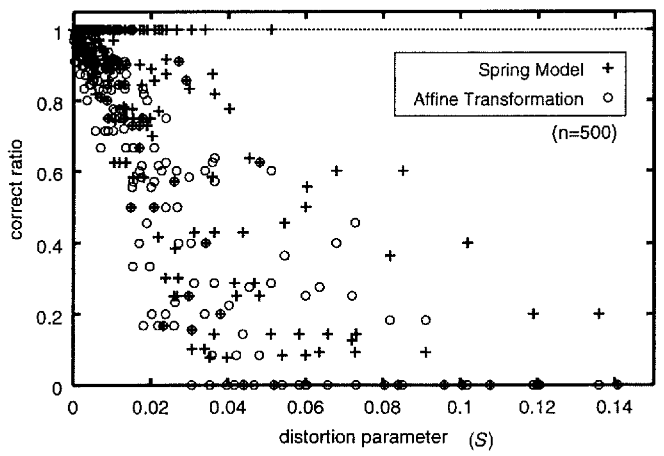

Fig. 9. Correct ratio with distortion for two new techniques.

Figures 10 and 11 show the relationship between the distortion parameter and number of particles. The solid line denotes the averaged correct ratio. With increasing the number of particles, the average distance between the particles decreased, resulting in the distortion effects to be small. However, as shown in these figures, there were almost no effects of the number of particles on the tracking effectiveness. The tendency of cross mark (n = 500) distribution was similar to that of square mark (n = 2000) distribution. The results indicates that the particle concentration ratio in the fluid had no direct effects on the tracking efficiency.

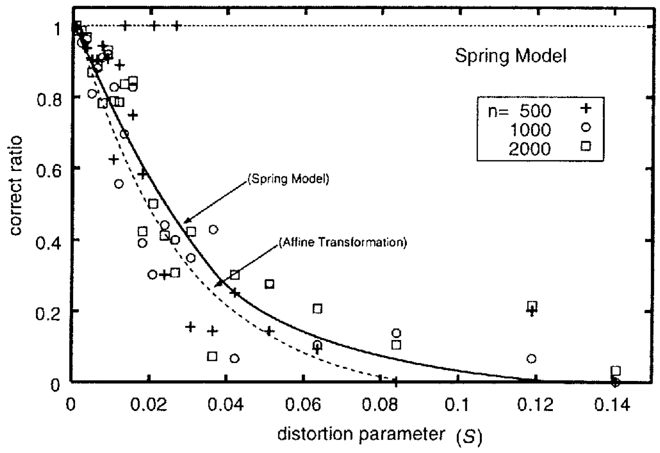

Fig. 10. Effects of particle concentration for Spring Model.

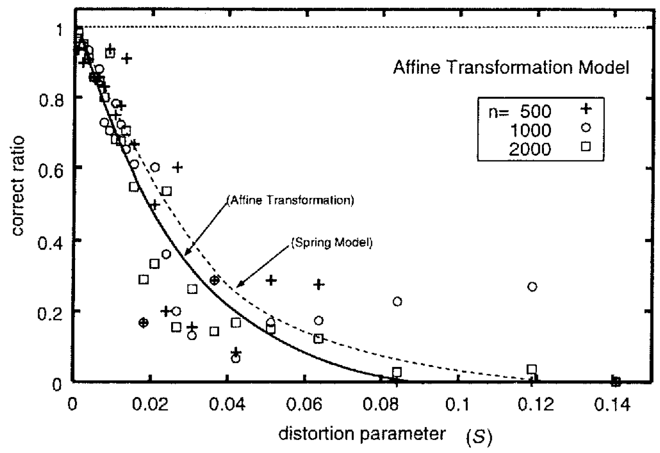

Fig. 11. Effects of particle concentration for Affine Transformation.

The CPU time for the Spring Model was about twice as long as that for the cross correlation. The CPU time for the Affine Transformation was a little bit longer than that for Spring Model. Because the concentration of the particle had little effects on the tracking efficiency, the 3D fields could be divided into small volume, in which the Spring Model and Affine Transformation technique were applied. By using the volume division technique, the CPU time for three-dimensional calculation could be shorten.

4. Conclusion

Tracking algorithms for three-dimensional Particle Image Velocimetry have been discussed, which can be applied to a flow field with rotation and shear. In the Affine Transformation technique, corresponding particle clusters between first and second image are determined. Assuming the particle pattern deformation to be expressed by the Affine Transformation, the pattern matching between the clusters is carried out.

The effectiveness of the tracking techniques, including 3D cross correlation, Spring Model and Affine Transformation, were evaluated with synthetic data of three-dimensional flow field.

The cross correlation technique could only be applicable to the small deformation cases. In the Affine Transformation technique, when the deformation of particle pattern between two images were very large, the pattern deformation could not be expressed by the Affine Transformation, i.e., linear transformation, resulting in the mis-tracking. However, the Spring Model technique was found to be more effective even in the larger deformation condition, because the Spring Model did not assume the linear transformation.

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